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New Spatial Correlation Assessment Procedure: application to Île de Ré Bridge data

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Abstract

Tomorrow's Megastructures exploitation will require robust management policies, including Non-Destructive Techniques measurements and Structure Health Monitoring. These inspections strategies already allow acquisition of large amounts of data at reasonable cost. However, the bigger the structure, the costlier the data collecting. This raises the question of spatial optimization, closely related to the determination of spatial variability. Related studies have so far characterized it using covariograms on raw data without paying attention to their conditions of validity. As this lack of rigor may lead to severe misestimating, we propose a mathematically rigorous spatial variability assessment procedure and apply it to measurements coming from the Île de Ré bridge diagnosis, performed during the DéCoF-Ré¹ project. This unique data set of resistivity and corrosion potential measurements allow us to begin a spatial variability database for marine concrete structures.

Keywords: spatial variability; piecewise-trend-stationarity; mean changepoints identification; maximum likelihood estimation; non-destructive techniques; concrete; bridges; inspection and maintenance.

¹ Décision par diagnostic de Corrosion en toute Fiabilité sur le pont de l'île de Ré

1 Introduction

Tomorrow's Megastructures exploitation will require strong and robust management policies, including Non-Destructive Techniques measurements and Structure Health Monitoring. These inspections strategies already allow acquisition of large amounts of material data on entire areas, at reasonable cost. However, the bigger the structure, the costlier the data collecting, which raises the question of spatial optimization. This concern is closely related to the determination of the spatial variability of quantities of interest [1,2], geostatistically characterized by their fluctuation parameters θ (Figure 1).

Nowadays, due to the lack of rich spatial studies on concrete megastructures, there is no reliable fluctuation parameter database, hence the need to perform estimations on a case-by-case basis. Related studies have so far determined this parameter using classic geostatistical tools such as covariograms and semi-variograms on raw measurements, without paying attention to their conditions of validity [2,3].

However, estimation based on unique samples Z (also called trajectories), as it is the case, by their very nature, on megastructures, imposes both their stationarity (constant mean) and ergodicity (spatial independence of highly spaced measurements). Indeed, if one of the hypotheses is not valid, interpretation based on these tools may not be relevant. It may even be hazardous for risk analysis applications, as they can lead to significantly underestimated probabilities of failure [4].

Yet, spatially variable environmental conditions, frequently encountered along megastructures,

induce non-continuous deterministic trends, i.e. non-stationarities.

This raises a crucial issue and motivates the development of the new Spatial Correlation Assessment Procedure that we present in this article.

In section 2, we set the mathematical framework of the problem and illustrate it with a simple case.

In section 3, we propose then a rigorous Spatial Correlation Assessment Procedure applicable to

Gaussian or assimilated Unidimensional Data, called SCAP-1D. We validate it with simulated sample data and gives its work range. Complete theoretical developments are available in the related journal article [5].

In section 4, we apply SCAP-1D to corrosion potential measurements performed on the Île de Ré Bridge as part of the DéCoF-Ré project. We focus on the wind-exposed and wind-sheltered faces of two healthy piers and highlight the effect of wind exposure on the spatial variability of corrosion potential.

2 Mathematical framework

2.1 Trajectories model

For the sake on simplicity, we focus on piecewise-trend stationary and uniformly distributed realizations of unidimensional Gaussian Random Field (GRF).

Let $\mathbf{Z} = (Z_i)_{i=1}^n$ the studied trajectory, georeferenced by $\mathbf{x} = (x_i)_{i=1}^n$. Z writes

$$Z_i = \mu(x_i) + \sigma G(x) \Leftrightarrow \mathbf{Z} = \boldsymbol{\mu} + \sigma \mathbf{G} \quad (1)$$

with $\boldsymbol{\mu}$ its mean and $\sigma \mathbf{G}$ its random part.

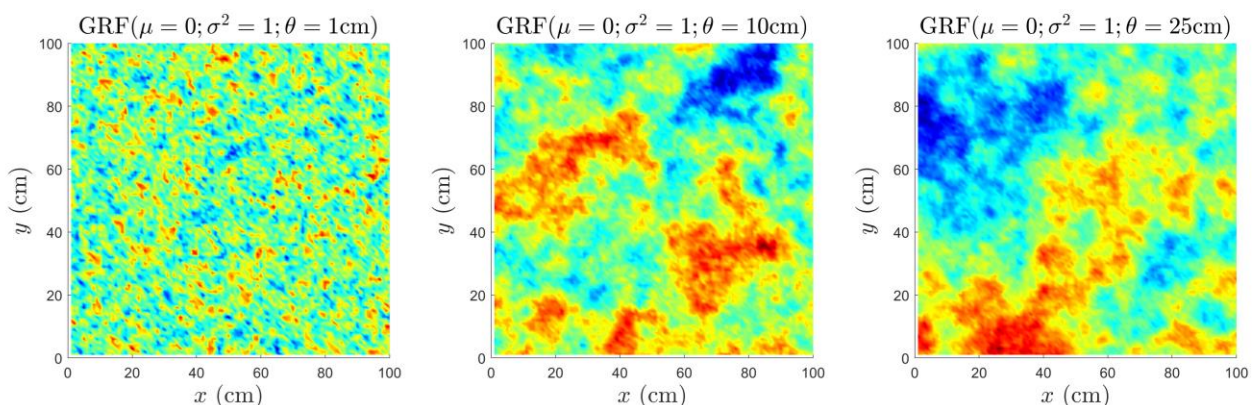


Figure 1. Impact of the fluctuation parameter θ on a Gaussian Random Field

The mean μ is considered piecewise-linear as this model can indeed simply fit numerous physical and environmental phenomena, such as discontinuities of realization, or exposition parameter evolution. Its discretized form writes

$$\boldsymbol{\mu} = \mathbf{A}\boldsymbol{\alpha} \quad (2)$$

with \mathbf{A} the nodal piecewise-linear function matrix on \mathbf{x} and $\boldsymbol{\alpha}$ the regression coefficients vector.

The random part is decomposed as the product of its constant standard deviation σ with \mathbf{G} , a realization of the standardized GRF \mathcal{G} of correlation function c_θ . We note $\mathbf{G} \sim \mathcal{G}(0,1, c_\theta)$.

c_θ defines the correlation $c_{i,j}$ between each couple (G_i, G_j) so that

$$c_{i,j} = c_\theta(\|x_i - x_j\|) = c_\theta(h) \in [0,1] \quad (3)$$

It depends on both their spacing h and the fluctuation parameter θ , which we aim to assess.

2.2 Classic estimation methods

2.2.1 Least-Square Estimation (LSE)

The fastest estimation method is Least-Square Estimation (LSE) based on the correlogram $\hat{c}(\mathbf{h})$ of \mathbf{Z} , which components write

$$\hat{c}(h) = \frac{1}{\hat{\sigma}^2 N_h} \sum_{(x_i, x_j) \in S_h} (Z_i - \hat{\mu}) \cdot (Z_j - \hat{\mu}) \quad (4)$$

with $\hat{\mu}$ and $\hat{\sigma}^2$ estimators of the mean and the variance, and N_h the cardinal of S_h , defined as the set of the couples (x_i, x_j) distant from h .

The covariance function is the theoretical model of the correlogram. Thus, assessing θ means fitting $\hat{c}(\mathbf{h})$ with a relevant model for $c_\theta(\mathbf{h})$, so that

$$\hat{\theta}_{LSE} = \operatorname{argmin}_\theta \left[\sum_h (\hat{c}(h) - c_\theta(h))^2 \right] \quad (5)$$

The most common, exponential correlation, writes

$$c_{\theta, \text{exp}}(h) = 1. \exp\left(-\frac{h}{\theta}\right) \quad (6)$$

Other popular models are presented in [6].

Note that, although implementing LSE is simple as no RF modelling is required, assessing estimators confidence regions (CR) may be difficult.

2.2.2 Maximum-Likelihood Estimation (MLE)

Contrary to LSE, Maximum-Likelihood Estimation (MLE) allows to easily get both accurate estimations and CRs. However, it requires to model the probability density function (pdf) f of the Random Field (RF).

Indeed, performing MLE of θ means maximizing its likelihood knowing \mathbf{Z} . This likelihood is equal to $f(\mathbf{Z})$ for RF with continuous pdf, which is the case of GRF $\mathcal{Z}(\mu, \sigma^2, c_\theta)$, so that

$$f(\mathbf{Z}) = \frac{1}{\sqrt{(2\pi)^N |\mathbf{C}_{\sigma, \theta}|}} \exp\left[-\frac{1}{2}(\mathbf{Z} - \boldsymbol{\mu})' \mathbf{C}_{\sigma, \theta}^{-1} (\mathbf{Z} - \boldsymbol{\mu})\right] \quad (7)$$

with $\mathbf{C}_{\sigma, \theta}$ the covariance matrix of \mathbf{Z} , defined as $\mathbf{C}_{\sigma, \theta, ij} = \sigma^2 c_\theta(x_i, x_j)$.

Thus,

$$\hat{\theta}_{MLE} = \operatorname{argmax}_\theta [f(\mathbf{Z})] \quad (8)$$

And by using the logarithm function, we get

$$\hat{\theta}_{MLE} = \operatorname{argmin}_\theta [\ln(|\mathbf{C}_{\sigma, \theta}|) + (\mathbf{Z} - \boldsymbol{\mu})' \mathbf{C}_{\sigma, \theta}^{-1} (\mathbf{Z} - \boldsymbol{\mu})] \quad (9)$$

2.2.3 Ergodicity Condition

It is worth mentioning that these estimation methods are meaningful only if studied trajectories come from ergodic RF, i.e. if assessment of these RF properties can be made on unique realizations.

Practically, we consider ergodicity when $\hat{c}(h)$ tends to zero within half the domain size. However, this hypothesis is rarely verified.

2.3 Problem of non-stationarity neglecting

In reality, when it comes to estimate the fluctuation parameter of a measured trajectory that could be modelled as a realization of a GRF, its mean is unknown and has to be beforehand estimated. This step is generally done using the classic statistic estimator $\hat{\mu}_{stat} = (1/n) \sum_i Z_i$.

However, it is relevant only if \mathcal{Z} is stationary, as it considers $\boldsymbol{\mu}$ constant. In the case of non-stationarity, $\hat{\mu}_{stat}$ is utterly biased, which raises a major issue as equations 4 and 9 show both LSE and MLE are based on precise estimation of the centred trajectory.

Illustration of the problem Figure 2 illustrates the problem with $Z \sim \mathcal{N}(\mu, \sigma, c_{\theta, \text{exp}})$, so that

$$\begin{cases} \forall x \in [0, 50[, \mu(x) = 5 \\ \forall x \in [50, 100], \mu(x) = 20 \end{cases} \quad (10)$$

Here, $\hat{\mu} = \hat{\mu}_{\text{stat}}$. Thus, $Z - \hat{\mu}$ is non-stationary and non-ergodic. Potential correlation length assessments based on $\hat{c}(Z - \hat{\mu}_{\text{stat}})$ are then false.

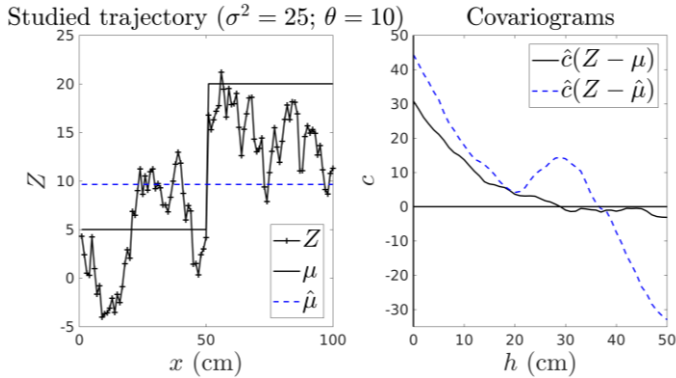


Figure 2. Illustration of the problem

3 The SCAP-1D procedure

3.1 Overview

In order to overcome this problem of wrong estimations in case of non-stationarities of Gaussian trajectories, we propose the following procedure: after detecting mean edges using the PELT algorithm [7], mean and fluctuation parameter are assessed via an iterative process we based on MLE, because of its interesting properties (2.2.2). This one is presented in the following. Then, while ergodicity is validated through analysis of the covariogram, stationarity and gaussianity hypotheses are verified using classic hypotheses tests on uncorrelated trajectory. Confirmation of these implies mathematically rigorous assessment of the geostatistical parameters. If not, any physical interpretation is rejected and model parameters have to be updated. These are the supposed number of mean edges, the mean regression degree, and the correlation function model. Procedure flowchart is illustrated on Figure 4 and whole related theoretical developments are fully available in [5].

3.2 Iterative MLE algorithm

Considering unknown mean and variance, the fluctuation parameter MLE writes

$$\hat{\theta}_{MLE} = \underset{\theta}{\operatorname{argmin}} [\ln(|\hat{C}_{\sigma, \theta}|) + (Z - \hat{\mu})' \hat{C}_{\sigma, \theta}^{-1} (Z - \hat{\mu})] \quad (11)$$

with $\hat{\mu} = A\hat{\alpha}$ (estimate form of Equation 2) and $\hat{C}_{\sigma, \theta, i, j} = \hat{\sigma} c_{\theta}(x_i, x_j)$. Hence the need to assess both the regression coefficients vector and the trajectory variance. Not neglecting spatial variability, associated MLE estimators write

$$\hat{\alpha}_{MLE} = \frac{Z' \cdot \hat{C}_{\sigma, \theta}^{-1} \cdot A}{A' \cdot \hat{C}_{\sigma, \theta}^{-1} \cdot A} \quad (12)$$

$$\hat{\sigma}_{MLE} = \frac{1}{n} (Z - \hat{\mu})' \hat{C}_{\sigma, \theta}^{-1} (Z - \hat{\mu}) \quad (13)$$

Looking at Equations 11, 12 and 13, it is clear that researched parameters are interdependent. That is why we use the following algorithm to perform their assessment (Figure 3)

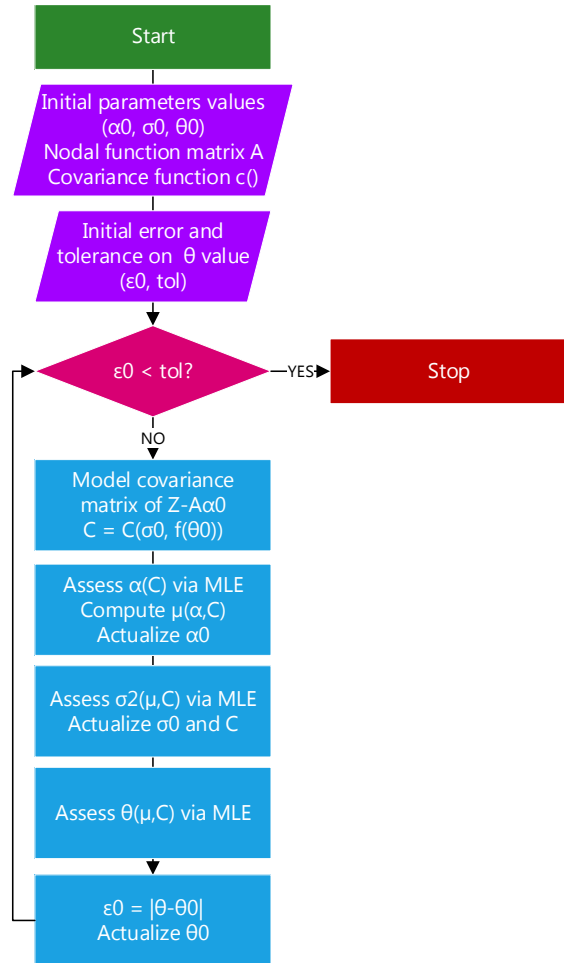


Figure 3. Iterative MLE algorithm of the procedure

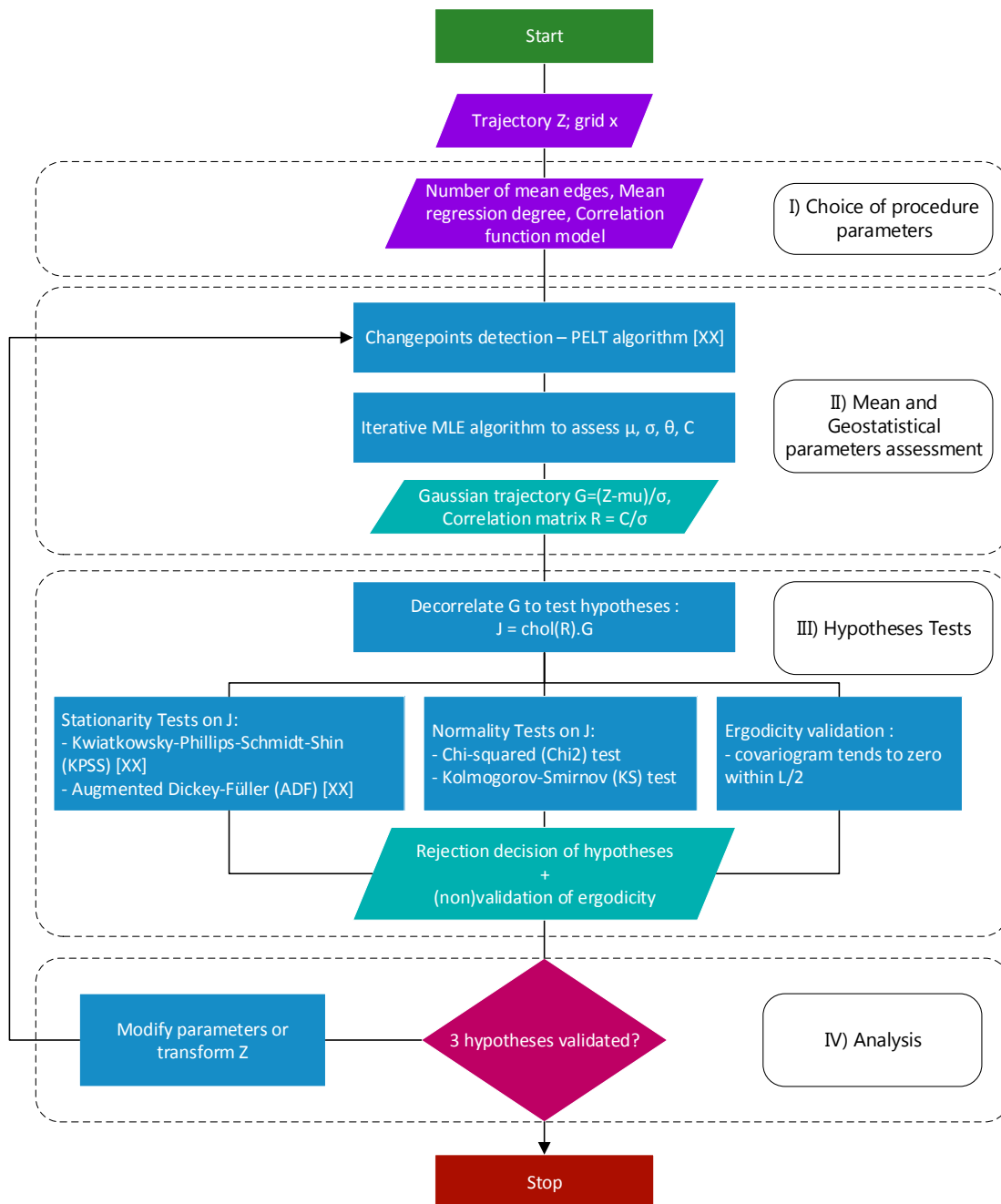


Figure 4. Flowchart of the SCAP-1D procedure

3.3 Numerical validation

3.3.1 Methodology

In [5], we test SCAP-1D for the three cases of (i) constant mean, (ii) mean step and (iii) mean slope step. In the first place, we simulate twenty trajectories $Z_k \sim N(5, 5, c_{10, \text{exp}})$ with the embedded circulant matrix method [8].

We keep 11 of them found to be indeed stationary, Gaussian and ergodic as test set for case (i).

Table 1 show estimation errors average and estimators Coefficients of Variations (CoV) compared to classic estimation method (based on covariogram LSE) ones.

It appears then that iterative MLE is more accurate for fluctuation parameter assessment.

Table 1. Comparison SCAP-1D – classic LSE

Method	Error - CoV μ	Error - CoV σ^2	Error - CoV θ
SCAP-1D	15% - 36%	23% - 10%	24% - 10%
Classic LSE	15% - 7%	22% - 10%	36% - \emptyset

Test set for case (ii) comprises 10 trajectories $Z_{6,sk}$ built from Z_6 by adding mean steps s between 0,1 to 10 times the standard deviation on $x = 49$. Figure 6 illustrates procedure execution on $Z_{6,s2}$.

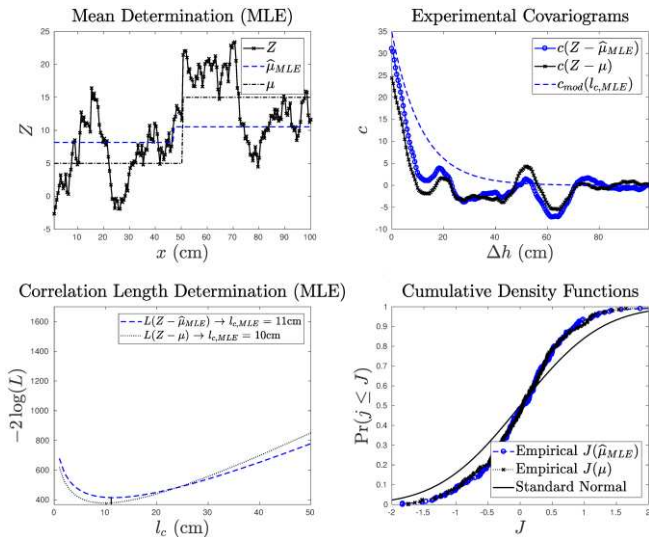


Figure 6. Application of SCAP-1D on $Z_{6,s2}$

Similarly, test set for case (iii) comprises 10 trajectories $Z_{6,bk}$ built from Z_6 by adding mean slope steps between 2 to 40 times the ratio σ/L . Figure 5 illustrates procedure execution on $Z_{6,b30}$.

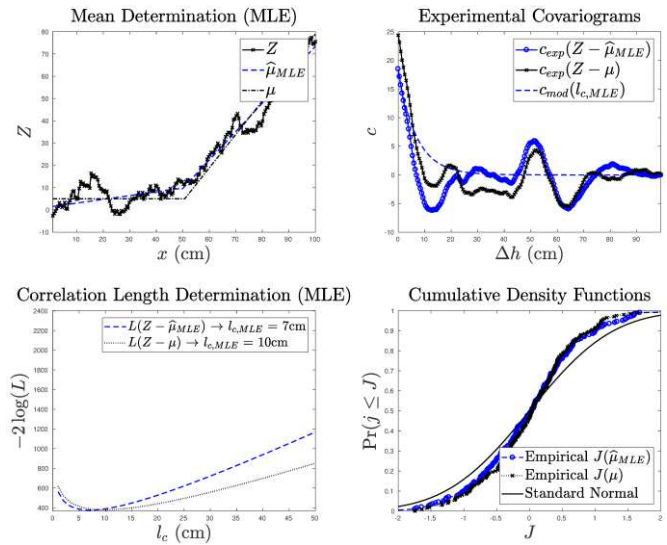


Figure 5. Application of SCAP-1D on $Z_{6,b30}$

3.3.2 Definition of work ranges

Numerical study show then that SCAP-1D performance depend on mean and slope steps values [5]. Results synthesis is made in Table 2.

In case (ii), SCAP-1D appears adapted to trajectories with mean steps above 2σ . Even if this step size is significant in classic statistical analysis, it is an interesting result as such a gap may not be obvious at first sight while working with spatial variability (Figure 6).

In case (iii), the procedure appears adapted to trajectories with slope steps above $30\sigma/L$. At this range, the error on parameters estimations is limited but higher than in case (ii). Thus, for potential multilinear cases, the different slopes have to be quite pronounced in order to use SCAP-1D and have confidence in estimations.

Table 2. Numerical validation synthesis (cases ii and iii)

step range / error %	x step	α_1	α_2	-	σ^2	θ	KPSS + ADF	KS + Chi2
0,1 – 2σ	7 - 98	16 - 63	21 - 60	-	2 - 40	1 – 25	Major No	Major Yes
$> 2\sigma$	0	7	1 - 3	-	2	0,5	Yes	Yes
slope step range / error %	x step	α_1	α_2	α_3	σ^2	θ	KPSS + ADF	KS + Chi2
2 - $30\sigma/L$	44 - 100	39 - 120	10 - 34	41 - 80	11 - 38	10 - 37	Major No	Major No
$> 30\sigma/L$	1 - 2	72 - 75	17 - 18	10 - 15	28	30	Yes	Yes

4 Application to Île de Ré Bridge corrosion potential measurements

4.1 Data set

Because of its geographical situation (marine environment with high tidal range $\sim 3.90\text{m}$), the Île de Ré bridge (Figure 7), made of concrete, is highly exposed to corrosion. Yet, it is a highly strategic structure. That is why the Charente-Maritime department involved in the DéCoF-Ré project, which aims to diagnose its piers corrosion state and to give decision help about scheduling and placement of future diagnostic operations.



Figure 7. The Île de Ré Bridge – Michel le Collen

In this context, we apply SCAP-1D on corrosion potential (E_{corr}) measurements made on piers horizontal rebars. The objective is to get information about fluctuation parameters associated to the couples piers - NDT.

4.2 Calculation approach

During DéCoF-Ré project, 15 piers (P_{xx}) have been inspected on their wind-exposed (FC) and wind-sheltered (FG) faces, separated in 3 one-meter-high zones: immersed(1), tidal(2), emerged(3).

Piers P_{13} and P_{23} have then been diagnosed healthy. Moreover, seasonality impact on their degradation indicators has been studied, resulting in four season-related data sets.

Thus, in order to focus on E_{corr} spatial variability characterization and to study the impact of wind-exposure on it, we apply SCAP-1D to zone 1 autumn data of the four faces. This allows to get rid of variations due to moisture and climate.

Studied trajectories are supposed to be realizations of GRF with exponential covariance. Table 3 synthetized fluctuation parameter estimations made on trajectories effectively found to be stationary, Gaussian and ergodic

4.3 Results and Interpretations

Primarily, several trajectories are non-ergodic, which may be due to side effects we neglected. We also point out trajectory $P_{13}FC(-0,03\text{m})$ is not intrinsically stationary and its mean is considered bilinear according to SCAP-1D (Figure 8).

Focusing on fluctuation parameters assessments, we note they lay between 10 and 30cm (5 to 15% of the domain size). We also point out sheltered faces present higher spatial variability than exposed ones, which appears relevant as chloride ions migrate faster in moist environments.

Moreover, P_{13} fluctuation parameters are higher than P_{23} ones. We can explain it by material properties differences as P_{23} concrete compressive strength is 5MPa higher. However, the correlation between this property and E_{corr} spatial variability has to be investigated.

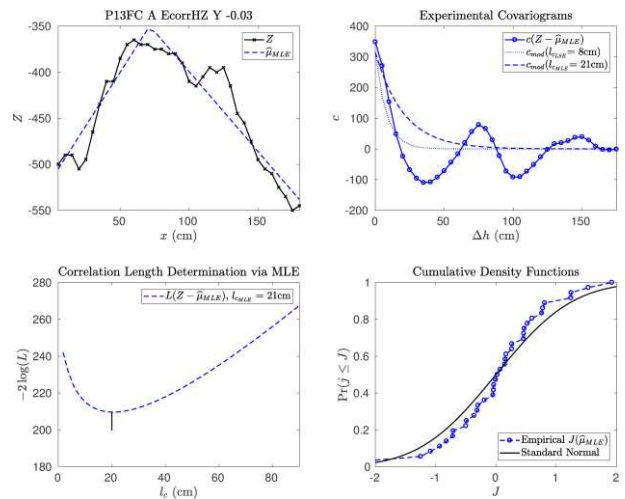


Figure 8. Corrosion potential at -0.03m on $P_{13}FC$

Table 3. Fluctuation parameters estimations

face	P13FC			P13FG		
$y(\text{m})$	0,84	0,65	0,41	-0,03	0,73	0,33
$\theta_{MLE}(\text{cm})$	19,3	34,27	36,19	20,51	15,49	20,42
face	P23FC		P23FG			
$y(\text{m})$	0,37	0,13	0,76	0,33		
$\theta_{MLE}(\text{cm})$	18,68	23,49	9,18	10		

5 Conclusions

In this paper, we proposed a rigorous spatial variability assessment procedure adapted to unidimensional piecewise-trend-stationary trajectories. This strategy may enable structure stakeholders to better optimize diagnosis campaigns. It may also be of great interest for design of Structure Health Monitoring systems.

While spatial variability is often estimated using approximate methods, our algorithm checks each required mathematical hypothesis while removing non-stationarities due to environmental effects, preventing calculation errors up to 100%.

Here, after a brief presentation of the theoretical background of SCAP-1D, numerical studies have been carried out to validate its ability to accurately estimate spatial variability of a Gaussian random field in three typical cases of a constant mean, a stepped-mean and a bilinear mean.

It has been shown that, (i) SCAP-1D precisely estimates the mean, variance and fluctuation parameter, (ii) the ratio step amplitude/standard deviation pilots its efficiency with a detection threshold around 2, (iii) the slope steps should be above 30 times the ratio standard deviation/domain size.

The procedure has finally been applied to corrosion potential measurements carried out on two piers of the Île de Ré bridge during the DéCoF-Ré project. It allowed to (i) demonstrate the robustness of the procedure, (ii) highlight the link between concrete moisture and corrosion potential spatial variability, and (iii) hypothesized correlation between its amplitude and concrete compressive strength.

In future works, we aim to extend the procedure to non-uniform and two-dimensional grids, allowing non-stationarities detection on whole surfaces without constraining measurements locations. We will also consider measurements uncertainties as it can strongly affect parameters estimation.

Moreover, SCAP-1D is to be applied to more data sets coming from the DéCoF-Ré project, as resistivity and corrosion potential have been measured on 15 piers and repeated during one year. It will allow us to propose substantiated reflections on evolution of the spatial variability of

degradation parameters in concrete megastructures.

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